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# Analyzing infeasible problems in AIM SMS

Marcel Hunting  
AIMMS Client Developer


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# Overview

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- Unusual Solver Termination
  - 🔍 Irreducible Infeasibility Set
  - 🔍 Excess Variables
  - 🔍 AIMMS Presolver
  - 🔍 Farkas Infeasibility Proof
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# Unusual Solver Termination

SOLVER	: CPLEX 12.6
Phase	: MIP
Iterations	: 19165
Nodes	: 1 (Left: 1)
Best LP Bound	: 833.8441312
Best Solution	: na
Solving Time	: 42.64 sec (Peak Mem: 25.9 Mb)
Program Status	: Intermediate infeasible
Solver Status	: Resource interrupt

Intermediate infeasible  
 Intermediate non optimal  
 Integer solution  
 No solution

Resource interrupt  
 Terminated by solver  
 Setup failure  
 Solver failure

- Check Messages Window (Ctrl+M)
- Check Solver log file

Option	Value
Solver listing messages	All
CONOPT 3.14V::status file display	Iteration log
CPLEX 12.6::mip display	Display each nth node
GUROBI 5.6::output file	Yes
KNITRO 9.0::status file display	Summary

# Irreducible Infeasibility Set (IIS)

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Definition:

An infeasible subset of constraints and variable bounds

  
small

Two options:

- Use solver functionality directly (CPLEX, Gurobi, XA)
- Use Math Program Inspector (Irreducible Inconsistent System)

Only available for LP and MIP (and quadratic problems)!

# IIS: Infeasibility Finder vs MPI

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Infeasibility Finder	Math Program Inspector
Uses algorithm in solver	Uses algorithm in AIMMS
Not available for CBC	Can be used with CBC
Also for QP/QCP/MIQP/MIQCP (CPLEX, Gurobi)	Only for LP and MIP
Special algorithm for MIP (CPLEX, Gurobi)	For MIP, applies IIS to RMIP

IIS cannot be used for NLP, MINLP, CP

# Adding Excess Variables

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$$\begin{array}{ll}
 \text{min/max} & \sum_j c_j x_j \\
 \text{subject to} & \sum_j a_j x_j \geq b_i \quad i \in I_1 \\
 & \sum_j a_j x_j \leq b_i \quad i \in I_2 \\
 & \sum_j a_j x_j = b_i \quad i \in I_3
 \end{array}$$

Feasibility problem:

$$\begin{array}{ll}
 \text{min} & \sum_{i \in I_1 \cup I_3} u_i + \sum_{i \in I_2 \cup I_3} v_i \\
 \text{subject to} & \sum_j a_j x_j + u_i \geq b_i \quad i \in I_1 \\
 & \sum_j a_j x_j - v_i \leq b_i \quad i \in I_2 \\
 & \sum_j a_j x_j + u_i - v_i = b_i \quad i \in I_3 \\
 & u \geq 0, v \geq 0
 \end{array}$$

# Excess Variables used by...

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- Math Program Inspector
  - Substructure Causing Infeasibility
- Violation penalties
  - Parameter with index domain *IndexVariablesConstraints*
- GMP::Instance::CreateFeasibility
  - Can also minimize maximum infeasibility



# AIMMS Presolver

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Bounds Tightening:

x: range [10,100]

y: range [5,8]

z: range [1,8]

Constraint:  $x + 4y - 2z = 60$



$$x = 60 - 4y + 2z \leq 60 - 20 + 16 = 56$$

$$x = 60 - 4y + 2z \geq 60 - 32 + 2 = 30$$

} x: range [30,56]

# Nonlinear Problems

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- **Local** nonlinear solver (CONOPT, KNITRO, SNOPT, IPOPT, AOA) might return **infeasible** while problem is feasible
- Approaches: AIMMS presolver, Violation penalties
- For an **infeasible** problem the **AIMMS presolver** often cannot find an infeasibility
- Another approach: Temporary remove **nonlinear constraints** and check whether resulting **linear problem** is infeasible (and then use IIS)

# Farkas Infeasibility Proof

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$u, v, w$ : nonnegative

$$\begin{array}{rcl}
 v + w & \geq & 2 \quad 0 \\
 u - 2v - w & \geq & 3 \quad 2/3 \\
 -2u + v - w & \geq & 2 \quad 1/3 \\
 \hline
 -v - w & \geq & 8/3 \quad +
 \end{array}$$

- Supported by CPLEX and Gurobi
- Only for LP (and RMIP)

# Farkas Infeasibility Proof

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$u, v, w$ : nonnegative

$$\begin{array}{rcl}
 v + w & \geq & 2 \quad 0 \\
 u - 2v - w & \geq & 3 \quad 2/3 \\
 2u - v + w & \leq & -2 \quad -1/3 \\
 \hline
 -v - w & \geq & 8/3 \quad +
 \end{array}$$

- Supported by CPLEX and Gurobi
- Only for LP (and RMIP)

# References

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- Language Reference (AIMMS 3.14)
  - Violation penalties: Chapter 15.4
  - AIMMS Presolver: Chapter 17.1
- AIMMS Help
  - Math Program Inspector
  - Infeasibility Finder
  - Farkas Infeasibility Proof
- Function Reference
  - `GMP::Instance::CreateFeasibility`
- AIMMS Blog
  - <http://blog.aimms.com/2013/03/dealing-with-model-is-infeasible-or-unbounded-error/>