Using AIMMS for Monte Carlo Simulation

AIMMS

Product Training Webinar

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November 16, 2016

Agenda

- Introduction
- Steps to do Monte Carlo simulation in AIMMS
- A demo of running multiple simulations in parallel on AIMMS PRO

A Not Rigid Classification of Analytical Models

	Decision Generation	Decision Evaluation
Certainty	• Linear Programming • Network Models • Integer & Mix-integer Programming • Nonlinear Programming • Control Theory	• Deterministic Simulation • Econometric Models • Systems of Simultaneous Equations • Input – output models
Uncertainty	• Decision Theory • Inventory Theory • Stochastic Control Theory • Stochastic Programming • Robust Optimization	• Monte Carlo Simulation • Econometric Models • Stochastic Processes • Queueing Theory • Reliability Theory

Reference: Bradley, Hax, and Magnanti - Applied Mathematical Programming

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A Not Rigid Classification of Analytical Models

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Uncertainty	• Decision Theory • Inventory Theory • Stochastic Control Theory • Stochastic Programming • Robust Optimization	• **Monte Carlo Simulation** • Econometric Models • Stochastic Processes • Queueing Theory • Reliability Theory

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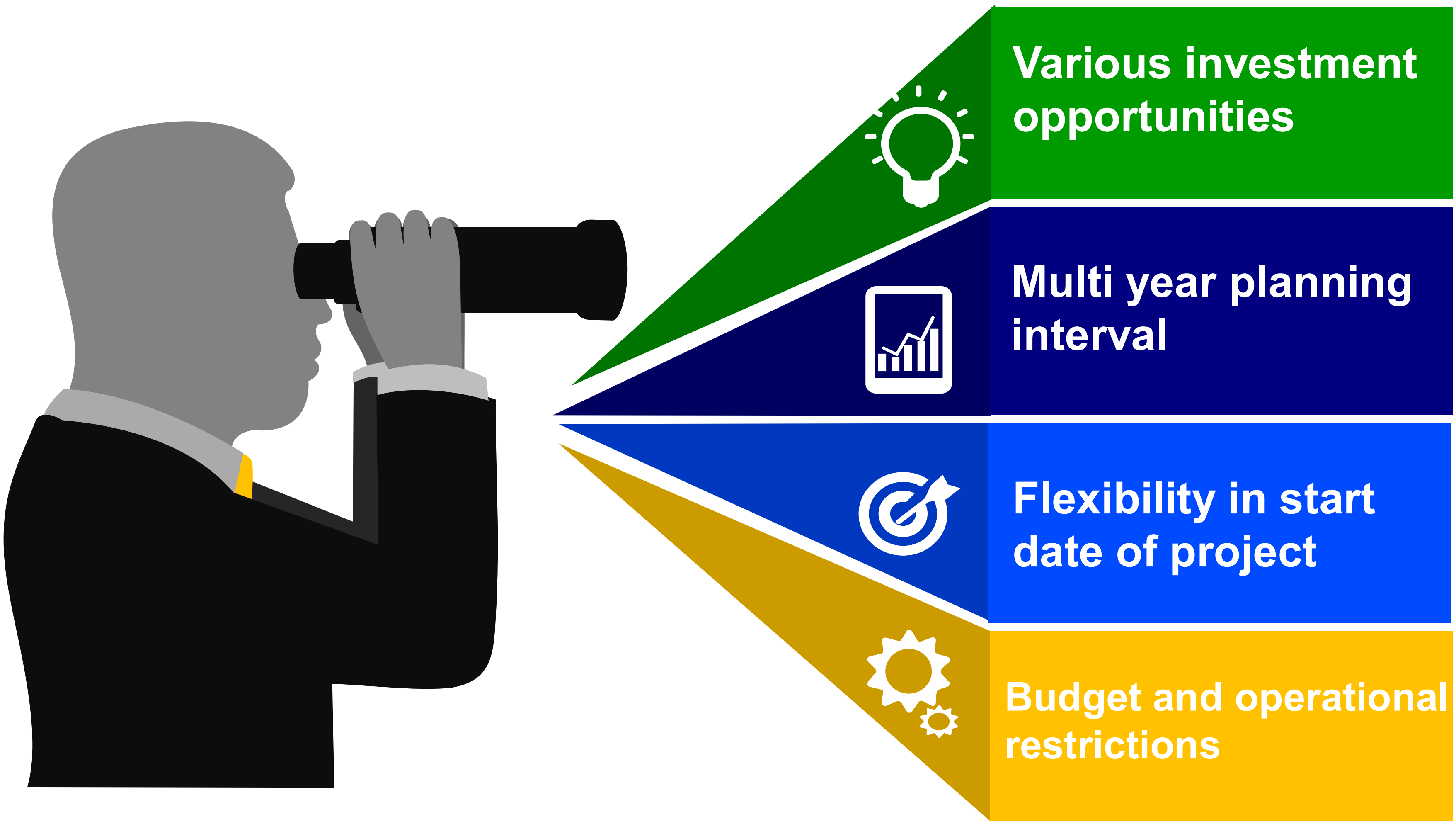
Monte Carlo Simulation - Steps

> Generate Decisions (LP/MIP/Stochastic Programming …)
> Sample uncertain input parameters
> Compute decision outcome under each sample
> Analyzing results
> Data visualization

Example – Project Planning



maximize the long term return

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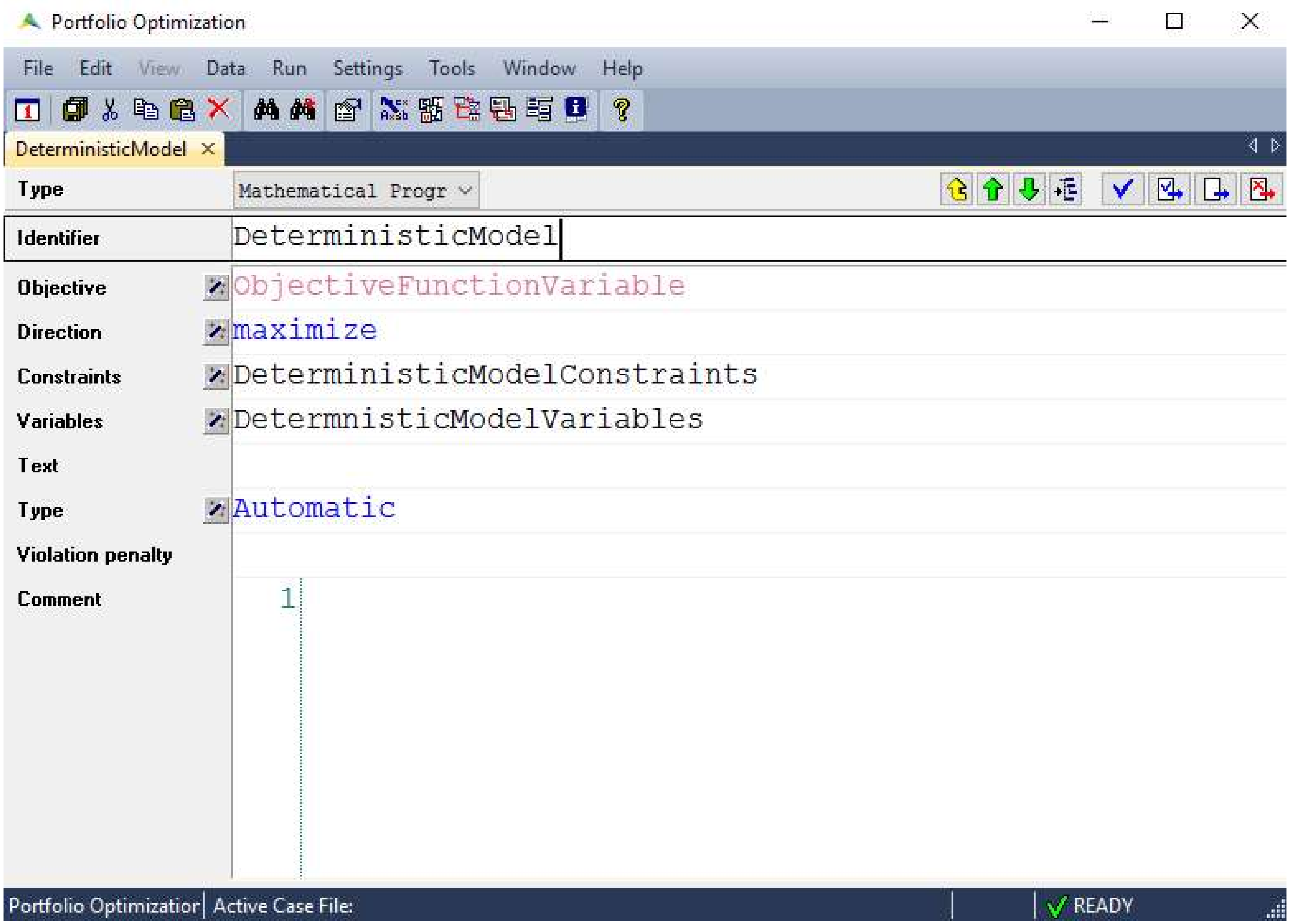
Optimization Model



Example – Project Planning

> The constructed model is a large (deterministic) MIP with estimation on

- Commodity pricing
- Capital expense
- Operating cost
- NPV discount factor

? How will this decision plan perform under the impact of uncertainty

? How will various possible decision plans perform under uncertainty

Monte Carlo Simulation - Steps

> Generate Decisions (LP/MIP/Stochastic Programming …)

> Sample uncertain input parameter

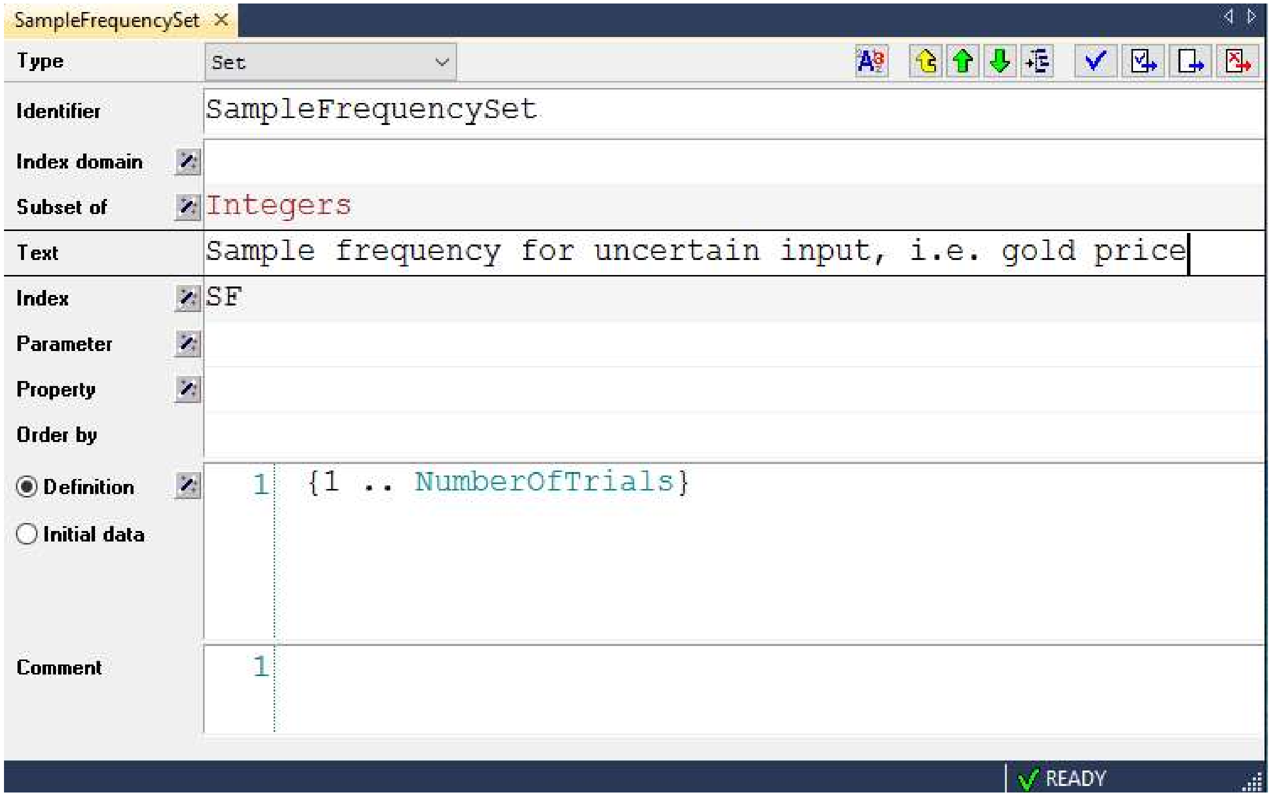
- **Declare sample set**

> Compute decision outcome under each sample

> Analyzing results

> Data visualization

Declare Sample Set



Monte Carlo Simulation - Steps

> Generate Decisions (LP/MIP/Stochastic Programming …)

> Sample uncertain input parameter

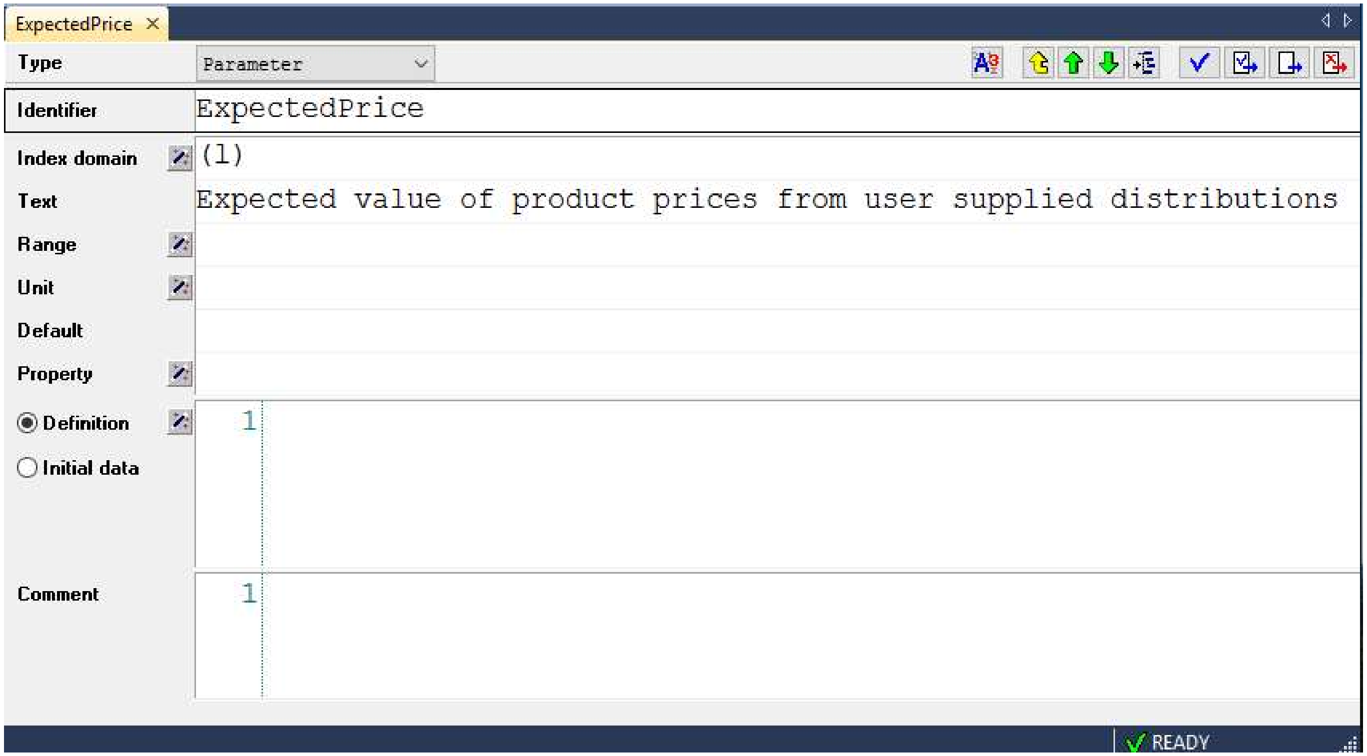
- Declare sample set
- **Extend uncertain parameter to sample space**

> Compute decision outcome under each sample

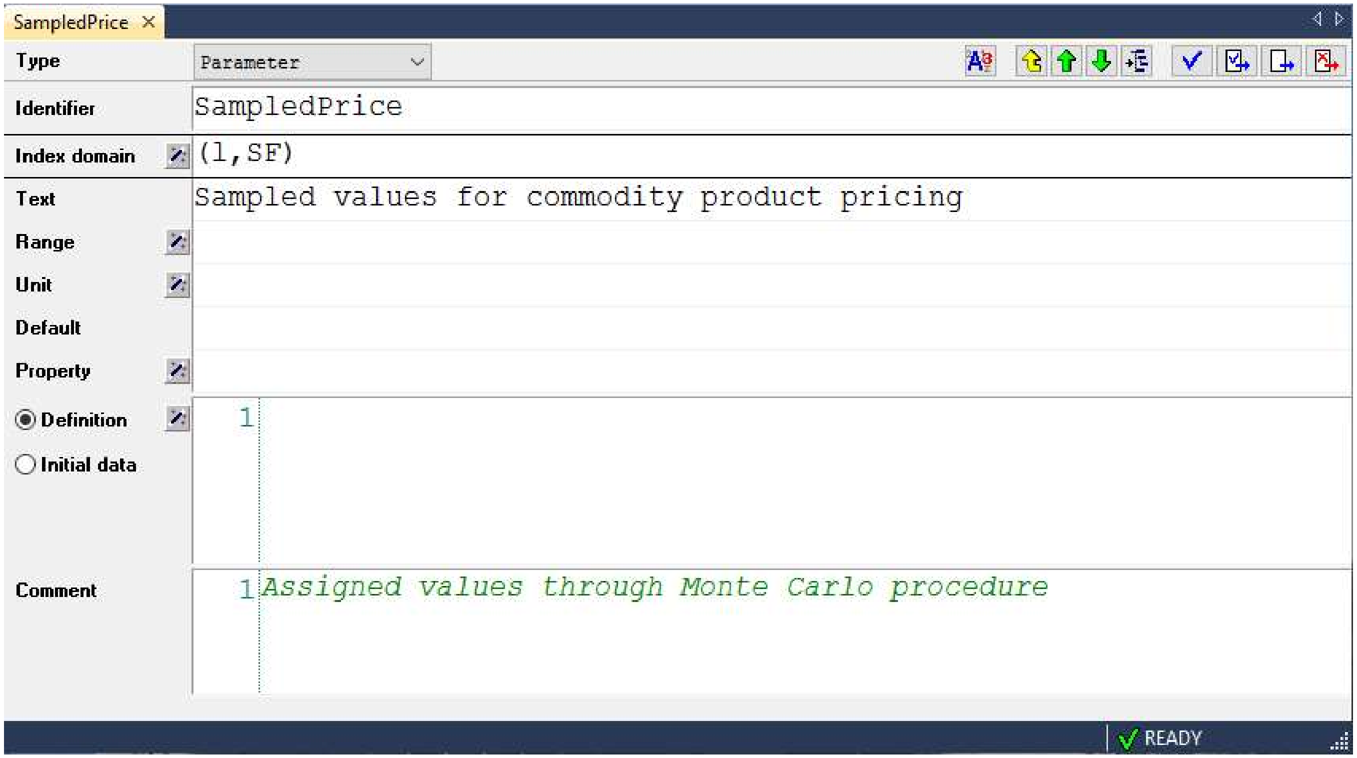
> Evaluate different outcomes occurring

> Data visualization

Extend Uncertain Parameter to Sample Space



Extend Uncertain Parameter to Sample Space



Monte Carlo Simulation - Steps

> Generate Decisions (LP/MIP/Stochastic Programming …)

> Sample uncertain input parameter

- Declare sample set
- Extend Uncertain Parameter to Sample Space
- **Generate sample data**

> Compute decision outcome under each sample

> Evaluate different outcomes occurring

> Data visualization

Distribution Functions in AIMMS

The functions for discrete distributions are:

- Binomial
- Geometric
- HyperGeometric
- NegativeBinomial
- Poisson

The functions for continuous distributions are:

- Beta
- Exponential
- ExtremeValue
- Gamma
- Logistic
- LogNormal
- Normal
- Pareto
- Triangular
- Uniform
- Weibull

Reference: AIMMS Function Reference

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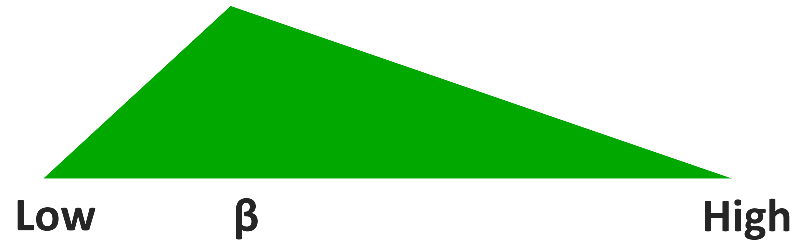
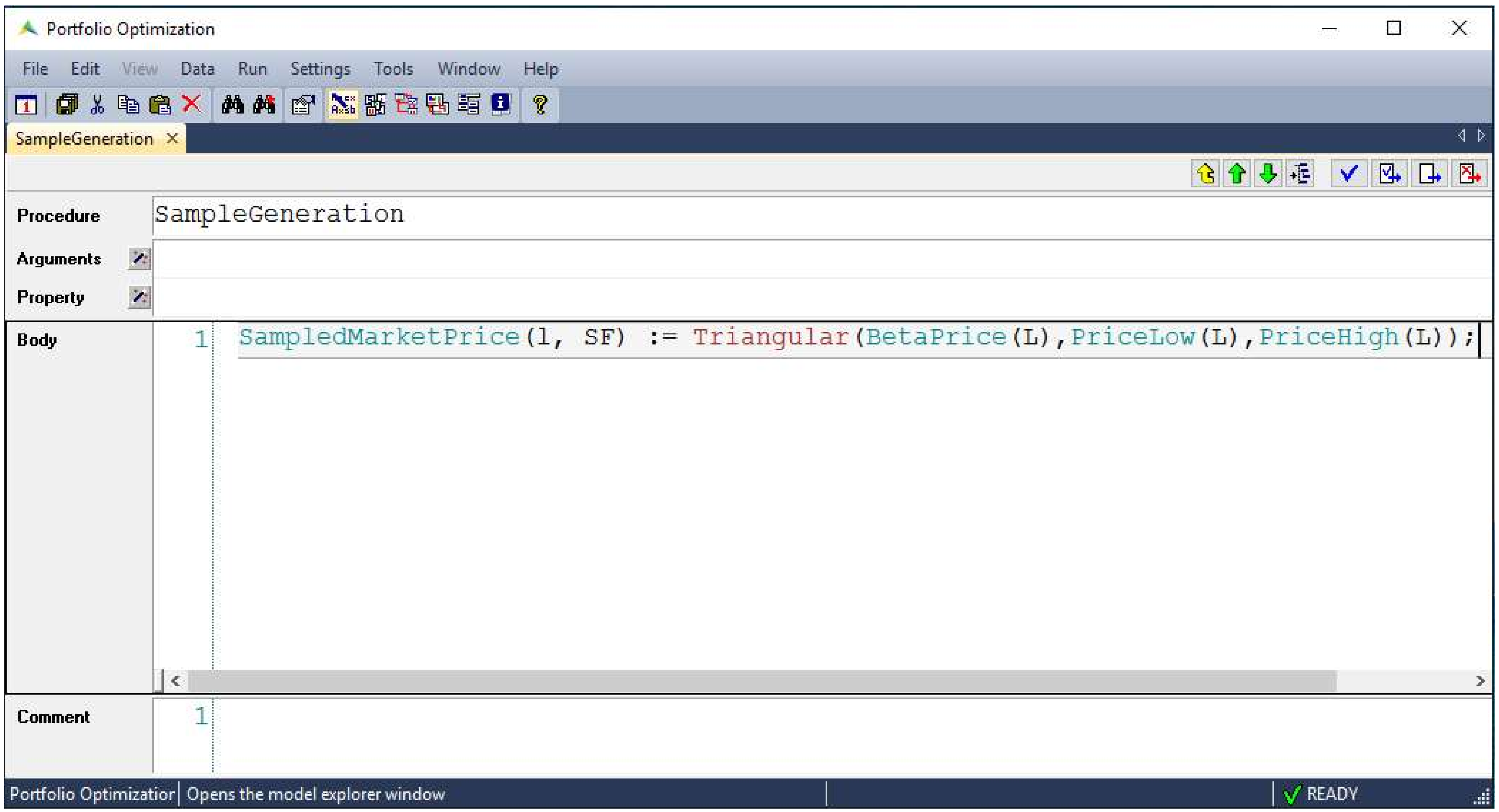
Generate sample data





View the results

Expected Price

	-
Phoenix	205,000
San Franc...	700,000
Los Angel...	530,000
Miami	210,000
New York	375,000
Chicago	205,000
Seattle	370,000
Santa Fe	284,000
Boston	350,000
Portland	289,900
Dallas	129,700

Sample Price

	1	2	3	4	5	6	7	8	9	10	11	12	13
Phoenix	162,714	204,515	195,043	182,198	236,850	211,902	162,881	277,207	231,311	246,496	181,440	245,268	201,
San Franc...	615,625	749,318	876,789	764,867	650,580	647,989	727,686	815,270	763,877	638,784	635,387	590,275	688,
Los Angel...	468,782	449,500	545,563	591,806	619,132	506,391	608,862	485,719	591,897	613,871	371,827	461,053	517,
Miami	174,237	175,673	205,299	221,225	200,769	184,421	218,322	168,305	213,963	228,715	172,892	209,303	151,
New York	369,759	374,448	380,525	377,149	512,207	329,870	531,983	505,951	472,635	339,344	456,300	360,808	271,
Chicago	183,550	226,983	194,800	156,128	243,243	211,402	139,511	196,693	225,789	190,208	179,236	199,989	160,
Seattle	383,904	431,635	313,401	404,990	232,234	309,690	420,095	460,099	392,532	367,858	369,853	390,666	342,
Santa Fe	219,903	230,815	345,223	269,201	301,607	299,982	286,882	227,740	280,789	315,190	191,996	261,786	300,
Boston	367,141	352,920	227,647	212,827	323,830	295,526	257,319	285,321	272,210	367,733	373,755	263,270	303,
Portland	352,996	289,743	313,445	324,357	264,798	313,880	301,248	299,282	290,725	297,047	400,410	259,231	357,
Dallas	134,503	145,740	87,521	117,055	142,210	96,560	152,565	124,215	154,464	132,330	116,777	106,368	120,

Monte Carlo Simulation - Steps

> Generate Decisions (LP/MIP/Stochastic Programming …)

> Sample uncertain input parameter

> Compute decision outcome under each sample

- Fix first stage decision
- Run the model under each sample data
- Store result

> Analyzing results

> Data visualization

Compute decision outcome under each sample

- Fix first stage decision
- Run the model under each sample data
- Store result

```
ProjectStart(L, T).nonvar := 1;
for SF do
    MarketPrice(L) := SampledMarketPrice(L, SF);
    solve SimulationModel;
    SimulatedObjective(SF) := SimulationModel.Objective;
endfor;
```

Monte Carlo Simulation - Steps

> Generate Decisions (LP/MIP/Stochastic Programming …)

> Sample uncertain input parameter

> Compute decision outcome under each sample

> Statistical analysis

- Mean
- Median
- Standard Deviation
- Percentile
- …

> Data visualization

AIMMS Statistical Functions

Name	# Expr.	Computes over all elements in the domain
Mean,	1	the (arithmetic) mean
GeometricMean	1	the geometric mean
HarmonicMean	1	the harmonic mean
RootMeanSquare	1	the root mean square
Median	1	the median
SampleDeviation	1	the standard deviation of a sample
PopulationDeviation	1	the standard deviation of a population
Skewness	1	the coefficient of skewness
Kurtosis	1	the coefficient of kurtosis
Correlation	2	the correlation coefficient
RankCorrelation	2	the rank correlation coefficient

Reference: AIMMS Language Reference

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Example

```
MeanNPV      := Mean(SF, SimulatedObjective(SF));
MedianNPV    := Median(SF, SimulatedObjective(SF));
MaxNPV       := Max(SF, SimulatedObjective(SF));
MinNPV       := Min(SF, SimulatedObjective(SF));
StdNPV       := SampleDeviation(SF, SimulatedObjective(SF));
```

Example - Percentile

```
! Sort objective in ascending order
SortedOrderOfTrials := Sort( SF, SimulatedObjective(SF) ) ;
P10_NPVOutputs :=
SimulatedObjective(Element(SortedOrderOfTrials, Ceil(0.1*NumberOfTrials))) ;
P90_NPVOutputs :=
SimulatedObjective(Element(SortedOrderOfTrials, Ceil(0.9*NumberOfTrials))) ;
```

Put together

RunSimulation

Procedure: RunSimulation

Arguments

Property

Body

```
ProjectStart(L, T).nonvar := 1;
for SF do
        MarketPrice(L)  := SampledMarketPrice(L, SF);
        solve SimulationModel;
        SimulatedObjective(SF)  := SimulationModel.Objective;
endfor;

MeanNPV          := Mean(SF, SimulatedObjective(SF));
MedianNPV        := Median(SF, SimulatedObjective(SF));
MaxNPV           := Max(SF, SimulatedObjective(SF));
MinNPV           := Min(SF, SimulatedObjective(SF));
StdNPV           := SampleDeviation(SF, SimulatedObjective(SF));

SortedOrderOfTrials := Sort( SF, SimulatedObjective(SF) ) ;  ! Sort objective in ascending order
P10_NPVOutputs := SimulatedObjective(Element(SortedOrderOfTrials, Ceil(0.1*NumberOfTrials))) ;
P90_NPVOutputs := SimulatedObjective(Element(SortedOrderOfTrials, Ceil(0.9*NumberOfTrials))) ;
```

Comment

READY

Monte Carlo Simulation - Steps

> Generate Decisions (LP/MIP/Stochastic Programming …)

> Sample uncertain input parameter

> Compute decision outcome under each sample

> Statistical analysis

> Data visualization

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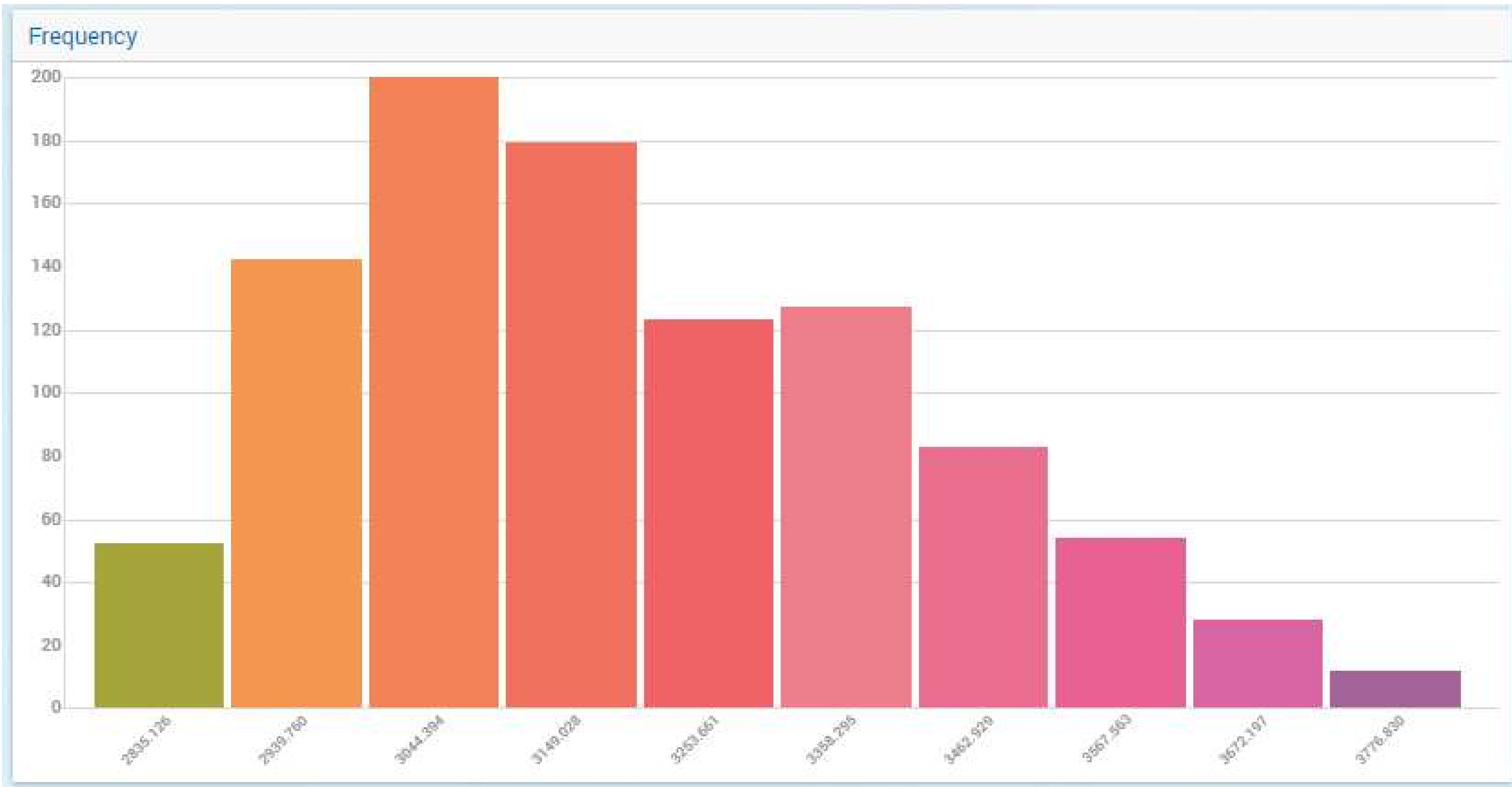
Histogram



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AIMMS Histogram Functions

AIMMS supports the following functions for creating and managing histograms:

- HistogramAddObservation
- HistogramCreate
- HistogramDelete
- HistogramGetAverage
- HistogramGetBounds
- HistogramGetDeviation
- HistogramGetFrequencies
- HistogramGetKurtosis
- HistogramGetObservationCount
- HistogramGetSkewness
- HistogramSetDomain

Reference: AIMMS Function Reference

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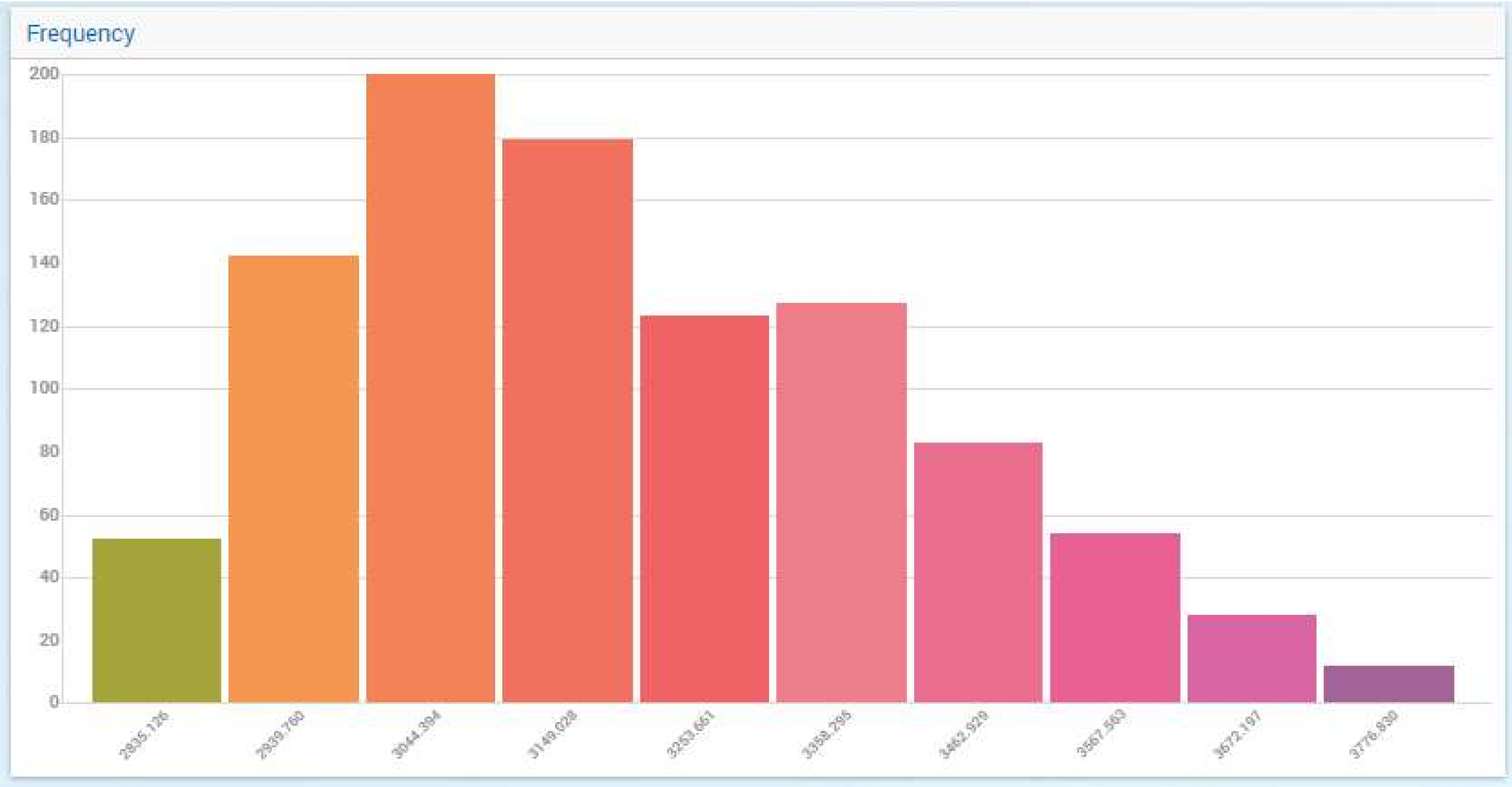
Example

```
HistogramCreate(HistogramID, sample_buffer_size: NumberOfTrials);
HistogramSetDomain(HistogramID, intervals: NumberOfHistogramIntervals,
left_tail : 0, right_tail : 0 ) ;

for sf do
    HistogramAddObservation( HistogramID,SimulatedObjective(SF)) ;
endfor;

HistogramGetFrequencies(HistogramID, HistogramFrequencies(HistInt) ) ;
HistogramGetBounds(HistogramID, HistogramLeftBounds(HistInt),
HistogramRightBounds(HistInt) ) ;
```

Show `HistogramFrequencies`



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How will this decision plan perform under the impact of uncertainty?

- > Generate Decisions (LP/MIP/Stochastic Programming …)
- > Sample uncertain input parameter
- > Compute decision outcome under each sample
- > Statistical Analysis
- > Data visualization

How will various possible decision plans perform under uncertainty?

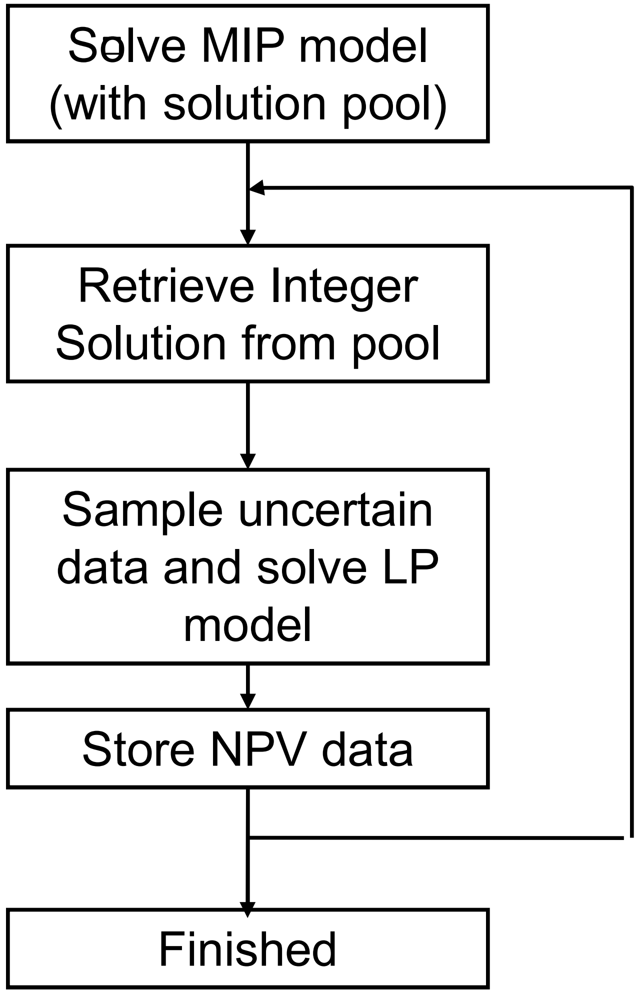
> Generate Multiple Decisions (LP/MIP/Stochastic Programming …)

> For each generated decision …

- Sample uncertain input parameter
- Compute decision outcome under each sample
- Statistical Analysis
- Data visualization

> Data Visualization

The Flow Chart



Generate Multiple Solution

> Use MIP Solution pool

> Use different initial estimation

> Use different model

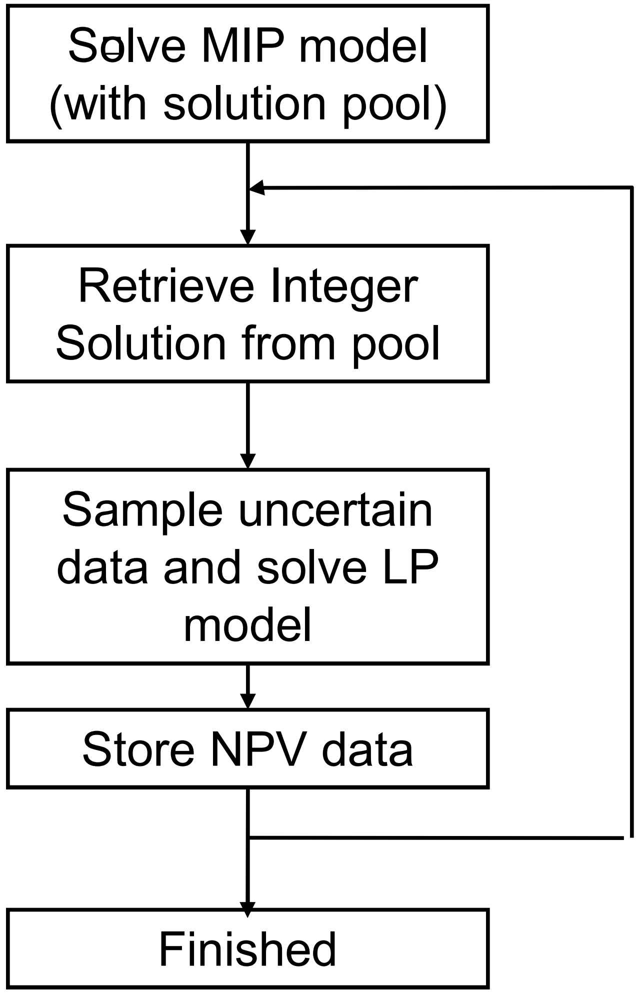
Example - MIP Solution Pool

```
! Procedure PROC_OptimizeMasterModel;

option do_populate := "Yes";
Option Pool_Replacement_Strategy := "Worst objective";
Option population_limit := SolutionPoolSize;
option Pool_Intensity := "Aggressive";
option Pool_Relative_Objective_Gap := 0.1;

...

GMP::Instance::Solve(DeterministicModel);

for( sc | ord(sc) <= NumberOfSolutions) do

    GMP::Solution::SendToModel(DeterministicModel,ord(sc));
    pool_ProjectStart (sc, l, t):= ProjectStart(l, t);

endfor;
```

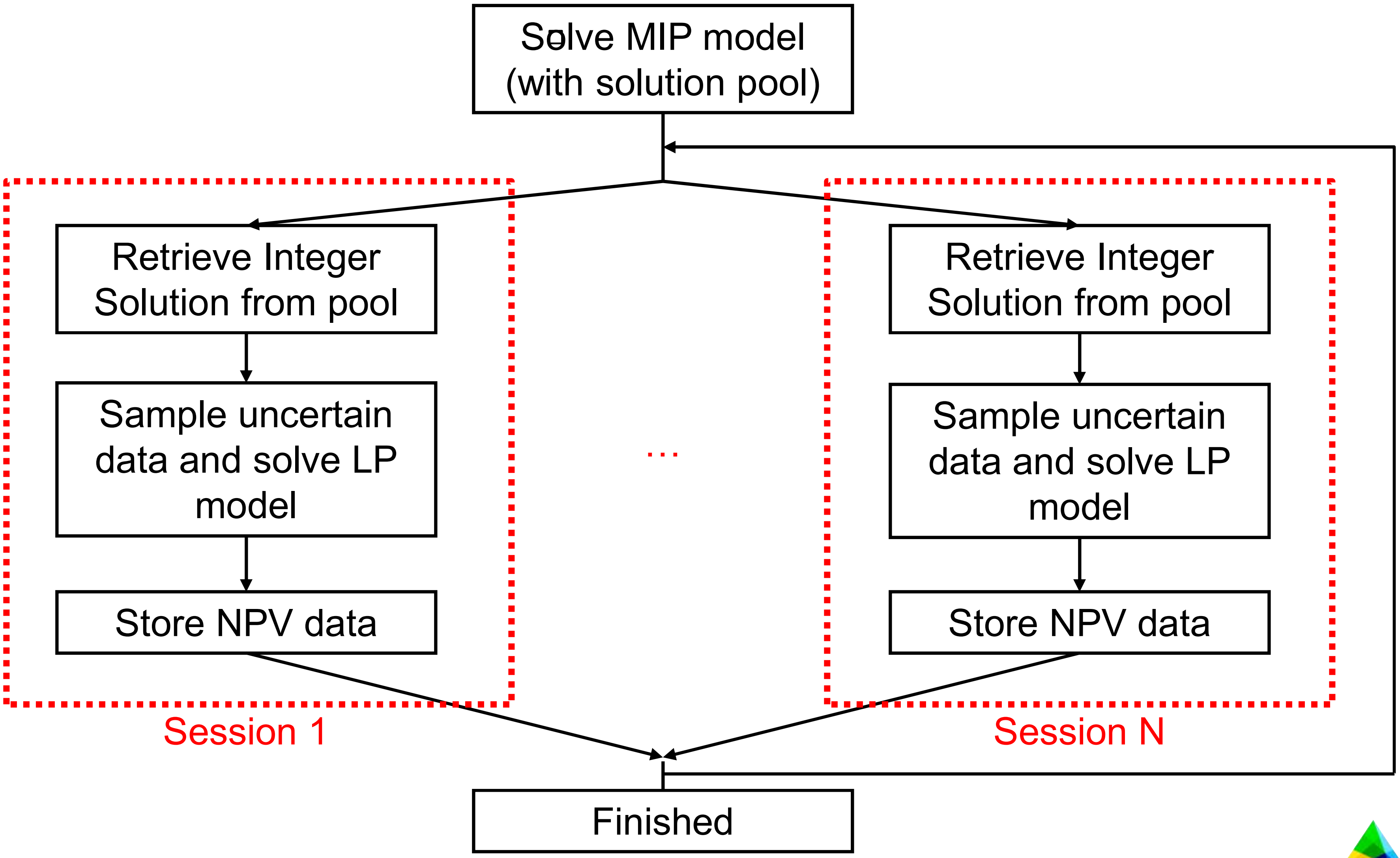
Example – Simulation

```
PROC_OptimizeMasterModel;


for( sc | ord(sc) <= NumberOfSolutions) do
    Selected_Scenario := sc;
    MonteCarloSimulationPRO(Selected_Scenario) ;
EndFor ;
```

The Flow Chart



Distributed Computing



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PROC_MonteCarloSimulationPRO(Selected_Scenario)

```
if pro::DelegateToServer(
    procedureName       :  'PROC_MonteCarloSimulationPRO',
    RequestDescription  :  RequestName_MC(Selected_Scenario),
    completionCallback  :  'PRO_LoadResults')
then return 1; endif;


ProjectStart(l,t):= pool_ProjectStart(Selected_Scenario,l,t);


PROC_MonteCarloSimulation;
```


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Data Visualization

More …

> AIMMS Forecasting Library

> Sample infeasibility check

> Model adjustable variables

> Data communication between client and PRO servers

Summary

- Distribution Functions
- Statistics Functions
- Histogram Functions
- Fix Variables
- Solution Pool
- AIMMS PRO DelegatetoServer
- AIMMS WebUI

Thank You

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Khang & Deanne